PROPERTIES OF REAL NUMBER

First recall all the given facts

***Factors**: Exact **divisors** of a number are called factors. A factor of a number is either less than or equal to the number.

Process for finding the number of different factors of a given Number

In general if $N = a^m b^n c^o d^p \ldots\ldots\ldots\ldots k^z$ where a,b,c,d,……….,k are **prime divisors** of N,then total number of all divisors of N = (m + 1)(n + 1)(o + 1)……………..(k + 1).

{ including 1 and itself(N)}

If 1 and N are excluded, then total number of proper divisors

= (m + 1)(n + 1)(o + 1)……………..(k + 1) - 2.

Ex. Find the total number of divisors of number- 4500.

solution: After doing prime factorisation of 4500, we get,

$4500 = 2^2\ 3^2\ 5^3$

Hence, total number of divisors = (2 + 1)(2 +1)(3 + 1) = (3)(3)(4)

= 36.

***Multiple:** Number obtained on multiplying a given number by any counting number is called its multiple.

The multiple of a number is either equal to or greater than the number.

Highest Common Factor(HCF) OR Greatest Common Divisor

A largest +ve integer that exactly divides given two or more than two +ve integers is called their Highest Common Factor(HCF).

METHODDS OF FINDING HCF:

I) BY PRIME FACTORISATION:

In this method we express each of the given numbers as the product of their prime factors. The product of the least powers of the common factors gives the HCF of the given numbers.

i.e. If two +ve integers a and b are written as $a = x^4 \; y^2$ and $b = x^2 \; y^3 \; z^4$; x, y, z are prime numbers, then HCF(a, b) = $x^2 \; y^2$

Ex.- The HCF of 120, 36 and 48

$120 = 2^3 \times 3 \times 5$

$36 = 2^2 \times 3^2$

$48 = 2^4 \times 3$

Here the common prime factors are 2 and 3 & their least power is 2 and 1 respectively.

Hence, the HCF of (120 , 36 and 48) = $2^2 \times 3 = 4 \times 3$

$= 12.$

II) By Continued division Method:-

In this method, we divide the larger number by the smaller number and get a remainder.

Divide the previous divisor by the remainder last obtained..

Repeat this until the remainder becomes zero.

The last divisor is the HCF of the given numbers.

First we divide 36 by 24

We get quotient as 1 and remainder as 12

Now divide previous divisor 24 by remainder 12

We get quotient as 2 and remainder becomes 0.

Hence, last divisor is 12

Therefore, the HCF(24 , 36) = 12.

III) To find the HCF of more then two number s, first find the HCF of any numbers and then find the HCF of the result and the third number and so on.

The final HCF is the required HCF.

EUCLID'S DIVISION LEMMA AND ALGORITHM

(DIVISIBILITY OF INTEGERS AND TECHNIQUE TO COMPUTE THE **HCF**)

EUCLID'S DIVISION LEMMA:

For any two +ve integers a and b, there exist two unique integers q and r satisfying

$$a = bq + r \text{ , where } 0 \le r < b.$$

Hare a = Dividend, b = Divisor, q = Quotient and r = Remainder

It can be written as,

Dividend = (Divisor X Quotient) + Remainder

Dividend

Divisor------b) a (q ----------Quotient

r ------------Remainder

****A lemma is a proven statement used for proving another statement.**

***<u>Questions from the applications of Division Lemma will be derive in school Class Room. So leave few questions at this stage</u>.**

EUCLID'S DIVISION ALGORITHM:

It is a technique to find the **HCF** of two +ve integers, this is based on Euclid's Division Lemma. According to this,

the HCF of any two +ve integers a and b, with a > b, is obtained as follow:

step I: Apply Euclid's division lemma to a and b to find q and r, where

$$a = bq + r, \quad 0 \le r < b.$$

Step II: If r = 0, the HCF is b. If r ≠ 0, apply Euclid's division lemma to b and r.

Step III: Continue the process till the remainder is zero.

The divisor at this stage will be HCF(a, b).

- Ex.- Find the HCF of 960 and 432.

Sol.- On applying Euclid's division lemma for 960 and 432,

we get 960 = (432 X 2) + 96

Hare, remainder = 96 ≠ 0,

so we take new dividend as 432 and divisor as 96.

Then, we get 432 = (96 X 4) + 48

Hare, remainder = 48 ≠ 0,

so we take new dividend as 96 and divisor as 48.

Then, we get 96 = (48 X 2) + 0

Here, the remainder is 0 and last divisor is 48.

Hence HCF of 960 and 432 is 48.

It is noticed that this method is same as **continue division method**, but in well-defined steps.

Hence, An algorithm means a series of well-defined steps which gives a procedure for solving a type of problems .

EXPRESSING THE HCF OF TWO NUMBERS AS A LINEAR COMBINATION OF GIVEN NUMBERS

To represent the HCF as a linear combination of the given numbers, we start from the last but one step and successively eliminate the previous remainder.

- Find the HCF of 960 and 432 and express it as a linear combination of 960 and 432

Sol.- $960 = (432 \times 2) + 96$ …………(i)

$432 = (96 \times 4) + 48$ ………….(ii)

$96 = (48 \times 2) + 0$ …………..(iii)

HCF(960 , 432) = 48.

Now, from eq^n (ii), we have

$48 = 432 – (96 \times 4)$

$= 432 – \{960 – (432 \times 2)\} \times 4$ [Substituting $96 = 960 – (432 \times 2)$ obtained from (i)]

$= 432 – 960 \times 4 + 432 \times 8$

$= 432 \times 9 – 960 \times 4$

$= 432 \times 9 + 960 \times (-4)$

$48 = 432x + 960y$, where $x = 9$ and $y = - 4$. Which is linear combination

It follow from the above example that HCF (let d) of two +ve integers a and b can be expressed as a linear combination of a and b i.e.,

$d = ax + by$; for some integers x and y

LEAST COMMON MULTIPLE(LCM)

LCM of two or more numbers is the smallest number which is divisible by all the given numbers.

Methods of finding LCM

i) Prime factorisation method:

Express each number as a product of prime factors then find Product of the each different prime factor involved in the numbers, with **highest power.**

i.e. If two +ve integers a and b are written as $a = x^4\ y^2$ and $b = x^2\ y^3\ z^4$; x, y, z are prime numbers, then LCM(a, b) = $x^4\ y^3\ z^4$

Ex.- The HCF of 120, 36 and 48

$120 = 2^3 \times 3 \times 5$

$36 = 2^2 \times 3^2$

$48 = 2^4 \times 3$

Here the different prime factors are 2,3 and 5 & their highest power is 4 ,2 and 1 respectively.

Hence, the LCM of (120 , 36 and 48) = $2^4 \times 3^2 \times 5 = 16 \times 9 \times 5$

$= 720.$

II) Common division Method:

Arrange the given numbers in a row in any order. Now divide by a prime number which divides exactly **at least two** of the given numbers carry forward the numbers which are not divisible. Repeat this process till no numbers have a common factor other than 1. **The product of the divisors and the remaining numbers is the LCM of the given numbers**.

***RELATION BETWEEN HCF AND LCM

1. The product of the HCF and LCM of **two numbers** is equal to the product of the numbers.

   HCF(a , b) X LCM(a , b) = a X b, where a and b are two numbers

2. Two numbers are co-prime then their HCF = 1 and hence,

   Product of the two given numbers = Their LCM.

3. The HCF always divides the LCM for a set of numbers.

   Hence, **HCF of numbers must be the factor of their LCM.**

4. HCF of given fractions = HCF of numerators / LCM of denominators

LCM of given fractions = LCM of numerators / HCF of denominators

**SOME IMPORTANT RESULTS FOR EASY TO SOLVE QUATIONS OF HCF & LCM

1. **The greatest number that will divide x , y and z leaving reminders a , b and c respectively is given by**

   [HCF of (x – a), (x – b),(x – c)]

2. **The greatest number that will divide x , y and z leaving the same reminder in each case is given by**

   [HCF of (x – y), (y – z),(z – x)]

3. The least number which when divided by x , y and z leaves the same remainder R in each case is given by

   [LCM of (x , y and z) + R]

4. The least number which when divided by x , y and z leaves the remainders a , b and c respectively is given by

   [LCM of (x , y and z) - P]

   where P = x – a = x – b = x – c

*** **FOR WORD PROBLEM : Read the QUESTION CAREFILLY and decide whether LCM or HCF of numbers required.**

SOME PROPERTIES OF NUMBERS

First recall

Prime Numbers: A number is called a prime number, if it has no factors other than 1 and the number itself.

2, 3 , 5 , 7 , 11, ….. are prime numbers.

****2 is the smallest and only even prime number.**

Composite Numbers: A number is called a composite number, if it has atleast one factor other than 1 and the number itself.

4, 6, 24, ………. are composite numbers.

****The smallest composite number is 4 and
Smallest composite odd number is 9.**

***** 1 is neither prime nor composite number.**

Co-Prime Numbers: Two numbers are co-prime if their HCF is 1.

HCF of 3 and 4 is 1

therefore 3 and 4 are co-prime numbers.

****1**. A composite number is express as the product of +ve integral powers of two or more prime numbers and this factorisation is unique except for the order in which the factors occur known as **FUNDAMENTAL THEROM OF AIRTHMATIC** given by mathematician **C.S. Gauss.**

$36 = 2^2 \times 3^2$ OR $36 = 3^2 \times 2^2$ both are same.

2. The digit at units place in an integer of the form a^n ; where a and n are natural number will be 0 if +ve integral powers of 2 and 5 (both) occur in the prime factorisation of the integer.**

3.The digits at units place in the integer a^n ; where a and n are natural number will be 5 if prime factorisation of a contains 5 and does not contain 2.**

PROVING A GIVEN NUMBER √a IS IRRATIONAL

First let given number √a be a rational number, then

$\sqrt{a} = p/q$, where HCF(p , q) = 1

Now, Cross-multiply and then squaring both sides, we get

$\sqrt{a} \times q = p$

$\Rightarrow \; aq^2 = p^2$ ----------------------------------(i)

then, use fundamental theorem of arithmetic according to which if prime number b divides c^2 , c being an integer , then b divides c.

=> a divides p^2

=> a divides p -----------------------------(ii)

then, let $p = ak$

$p^2 = a^2 k^2$ (by squaring)

$aq^2 = a^2 k^2$ [from (i)]

$\Rightarrow q^2 = a k^2$

=> a divides q^2

=> a divides q --------------------(iii)

Finally, from (ii) and (iii) we get a common factor other than 1 in p and q, that is a which contradict our assumption.

Thus our assumption that √a is a rational number is false.

Hence, √a is an irrational number.

****EXAMINE WHETHER A RATIONAL NUMBER HAS TERMINATING OR NON-TERMINATING REPEATING DECIMAL EXPAINTION**

If denominator q has a prime factor other than 2 and 5 then rational number (p/q) has a non-terminating repeating decimal expansion otherwise (if q is the form 2^m 5^n ; where m , n being +ve integers) it has terminating decimal expansion.

Ex. $23 / (2^3\ 5^2)$ is a terminating decimal expansion

$23 / (2^3\ 3^2)$ is a non- terminating repeating

****TO EXPRESS DECIMAL EXPANSION OF p/q**

If q is of the form 2^m 5^n ; where m , n being +ve integers

to make m = n , multiply numerator and denominator by 2 or 5 with suitable power

Then denominator convert into some power of 10

Now, we can easily written down decimal expansion of p/q

Ex. 15 / 1600

$= (3 \text{ X } 5) / (2^6 \text{ X } 5^2) = 3 / (2^6 \text{ X } 5)$

Now multiply numerator and denominator by 5^5 , to make same power of 2 and 5

$= (3 \text{ X } 5^5) / (2^6 \text{ X } 5 \text{ X } 5^5)$

$= (3 \text{ X } 3125) / (2^6 \text{ X } 5^6)$

$= (9375) / (10)^6$

$= (9375) / 1000000$

= 0. 009375 which is the required decimal expansion

***HCF AND LCM OF ALGEBRAIC EXPRESSION

For finding HCF and LCM of two or more than two algebraic expressions we produce same process as for finding HCF and LCM of numbers by factorisation method here first factorize all given algebraic expression, then

HCF = Multiplication of common factors with least power

LCM = Multiplication of different factors with higher power.

Ex.- Find HCF and LCM of

$p(x) = 2x^3 - 16$, $g(x) = x^2 - 4x + 4$, & $q(x) = x^2 - 3x + 2$

$p(x) = 2x^3 - 16$

$= 2(x^3 - 8)$

$= 2\{(x)^3 - (2)^3\}$

{by $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$}

$= 2(x - 2)(x^2 + 2x + 2^2)$

$= 2(x - 2)(x^2 + 2x + 4)$

$g(x) = x^2 - 2(x)(2) + 2^2$

{ by $a^2 - 2ab + b^2 = (a - b)^2$}

$= (x - 2)^2$

$q(x) = x^2 - 2x - x + 2$

$= x(x - 2) - 1(x - 2)$

$= (x - 2)(x - 1)$

HCF { p(x), g(x) & q(x) } $= (x - 2)$

and LCM { p(x), g(x) & q(x) } $= 2(x - 2)^2(x - 1)\,)(x^2 + 2x + 4)$

NOW, WE CAN EASILY SIMPLIFY OF ALGEBRAIC FRACTIONS ALSO

*******NOTED THAT NUMBER SYSTEM, POLYNOMIAL, ALGERAIC IDENTITIES,FACTORISATION PLAY VITAL ROLE IN MATHEMATICS. WHO IS SAYING ………………………………………………………. MANISH KUMAR DUTTA***

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